A NEW DESIGN METHOD FOR MAXIMUM GAIN FORMULATION OF A MICROWAVE AMPLIFIER SUBJECT TO NOISE FIGURE AND INPUT VSWR

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ABSTRACT- This paper presents a graphic design method for the low-noise, low input VSWR amplifiers, where all necessary design information is placed in the input impedance plane. As a consequence of the bilinear tranformations involved, all parameters can be represented by circles which centers and radii are in the input impedance plane. For a given set of the noise figure and input VSWR the maximum achievable gain and corresponding terminations can be determined by inspection of the graph. Furthermore not only the analytic expressions make the calculations very fast but the results of changes in the noise figure, input VSWR and gain can be viewed directly.

1. DESCRIPTION OF THE PROBLEM:

Noise figure of a linear noisy two-port with an arbitrary source impedance (Z_s) is expressed in terms of equivalent noise resistance (R_N), minimum noise figure as ratio (F_M) and optimum source impedance (Z_{op}) of the two-port, as follows: /1/

$$F = F_M + \frac{R_N}{|Z_{op}|^2}\,\frac{|Z_s - Z_{op}|^2}{R_s} \qquad (1)$$

$R_s = \mathrm{Real}[Z_s]$

Input VSWR of the two-port is a function of source (Z_s) and load (Z_L) impedance via input impedance (Z_1) of the two-port as follows: /2/

$$V_1 = \frac{1+|\rho_1|}{1-|\rho_1|}, \text{ where } |\rho_1| = \frac{|Z_s - Z_1^*|}{|Z_s + Z_1|};$$

$$Z_1 = z_{11} - \frac{z_{12}z_{21}}{z_{22}+Z_L} \qquad (2)$$

$z_{11}, z_{12}, z_{21}, z_{22}$ are small-signal open-circuit z-parameters of the two-ports and ρ_1 is the input reflection coefficient.

The transducer power gain of the same two-port is a function of the source and load impedances and the z-parameters are as follows:

$$G_T = \frac{4R_sR_L|z_{21}|^2}{|(z_{11}+Z_s)(z_{22}+Z_L)-z_{12}z_{21}|^2}$$

where $Z_s = R_s + jX_s$,

$$Z_L = R_L + jX_L. \qquad (3)$$

The problem can be described as a mathematically constrained maximization problem which is to find the maximum value of $G_T(R_s, X_s, R_L, X_L)$ subject to $\delta_1 = F_{req} - F(R_s, X_s) = 0$ and $\delta_2 = V_{1req} - V_1(R_s, X_s, R_L, X_1) = 0$ and the corresponding values of $Z_s = R_s + jX_s$, $Z_L = R_L + jX_L$ where F_{req} and V_{1req} are the required noise and input VSWR respectively.

Although the Lagrange multipliers method is suggested particularly for this equality constrained type of maximization problem, the equations obtained by differentiating the composite function $E = G_T + \lambda_1\delta_1 + \lambda_2\delta_2$ are high order algebraic polynomials which make it difficult to find an analytic expression for a solution of the problem. Instead of this mathematical solution a geometrical search has been carried out based on constant noise, input VSWR and gain circles in the source and input planes keeping the solution within the physical bounds.

2. CONSTANT NOISE, INPUT VSWR AND GAIN CIRCLES IN SOURCE IMPEDANCE (Z_S) PLANE.

A general circle equation in the Z_s plane whose center and radius are $Z_c = R_c + jX_c$ and r may be expressed as

$$|Z_s|^2 - 2R_cR_s - 2X_cX_s + |Z_c|^2 - r^2 = 0 \qquad (4)$$

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So, using (1) the equation of the circle family of the constant noise figure in the Z_s plane is

$$|Z_s|^2 - 2(R_{op}+N)R_s - 2X_{op}X_s + |Z_{op}|^2 = 0 \quad (5.1)$$

where $Z_{op} = R_{op}+jX_{op}$,

$$N = (F_{req} - F_M)|Z_{op}|^2/2R_N \quad (5.2)$$

Similarly , using (2) and (3) the equations of the constant VSWR and gain circle families can be respectively given as follows:

$$|Z_s|^2 - 2R_1 \frac{1+|\rho_1|^2}{1-|\rho_1|^2} R_s + 2X_1X_s + |Z_1|^2 = 0 \quad (6)$$

$$|Z_s|^2 - 2(C/G_T - R_i)R_s + 2X_1X_s + |Z_1|^2 = 0 \quad (7.1)$$

$$\text{where } C = 2R_L|Z_{21}|^2 / |Z_{22}+Z_L|^2 \quad (7.2)$$

The centre and radii of these circles can be easily determined by using the equation (1) , which will be denoted respectively as $Z_{cp} = R_{cp}+jX_{cp}$, $Z_{cv} = R_{cv}+jX_{cv}$ and r_{cp}, r_{cv} later. As is understood from (6) and (7.1), the centre of the VSWR and gain circles lie on the same imaginary axis which is $-X_1$. It can be shown that $r_p = r_v$ when $R_{cp} = R_{cv}$. The physical meaning of this is that only source impedance to be chosen on the required VSWR circle will always make gain maximum under the constrained of the required VSWR , which can be shown to be

$$G_T = \frac{|Z_{12}|^2}{|Z_{22}+Z_L|^2} \frac{R_L}{R_1} (1-|\rho_1|^2) \quad (8)$$

So only the required noise and VSWR circles can be taken into account in the Z_s plane , because any mutual point of these two circles not only satisfies both noise and VSWR requirements but also makes the gain maximum in the Z_s plane.

3. CONTROL OF THE POSITINS OF THE VSWR CIRCLES w.r.t. THE NOISE CIRCLES IN THE Z_s PLANE FROM THE INPUT IMPEDANCE PLANE

As can be seen from (5.1) and (6) the required noise circle is constant while the required VSWR circle can travel depending on load impedance via the input impedance , Z_1. Therefore , the following situations are possible. These circles may not touch , they become tangent or they cut each other. Since the tangent positions-external and internal positions- are the transitions stages , so they will be investigated which general equation in the Z_s plane as follows:

$$|Z_{cn} - Z_{cv}|^2 = (r_n \pm r_v)^2 \quad (9)$$

By using the expessions for Z_{cn}, Z_{cv}, r_n, r_v, the corresponding circle of this in the Z_1-plane is

$$|Z_1|^2 - 2[(R_{op}+N)\frac{1+|\rho_1|^2}{1-|\rho_1|^2} \pm \sqrt{N(N+2R_{op})} \frac{|\rho_1|}{1-|\rho_1|^2}]R_1 + 2X_{op}X_1 + |Z_{op}|^2 = 0 \quad (10)$$

which represents two different circles T_1 and T_2, because of two different tangent positions of the noise and input VSWR circles in the Z_s plane. It can be shown that the center of these T_1 and T_2 lies on the same imaginary axis and the circle T_2 always takes place inside the circle T_1 without touching. (Fig. 1) Five different regions in the input impedance plane bounded by T_1 and T_2 circles cause different interactions of the required VSWR circle with the noise circle in the Z_s plane. As is seen from Fig. 1, the regions numbered by 1 and 5 in the Z_1 plane do not cause any mutual point of both the noise and VSWR circles in the Z plane, so these cases do not give any solution.

4. GEOMETRICAL AND ANALYTICAL SOLUTION OF THE CONSTRAINED MAXIMUM GAIN IN THE Z_1-PLANE; DETERMINATIONS OF THE TERMINATIONS

To find the exact geometrical and analytical solution of this constrained problem, the constant gain circles given by eq. (8) are constructed in the Z_1 plane and then investigations of the value of these gain circles in the solution regions are made.

The equation of the circle family in the Z_1 plane with G_T as a parameter is found as follows:

$$|Z_1|^2 - (2r_{11}r_{22} - r - \frac{G_T|Z_{12}|^2}{1-|\rho_1|^2}) \frac{R_1}{r_{22}}$$

$$- \frac{X_1}{r_{22}}(2x_{11}r_{22}-x) - \frac{1}{r_{22}}(r\, r_{11}+x\, x_{11})$$

$$+|z_{11}|^2=0 \quad ; \text{ where } r_{ij}=\mathrm{Real}\{z_{ij}\} \ ,$$

$$x_{ij}=\mathrm{Imag}\{z_{ij}\}; \quad z=z_{12}z_{21}=r+jx,$$

$$r=\mathrm{Real}\{z_{12}z_{21}\} \ , x=\mathrm{Imag}\{z_{12}z_{21}\} \qquad (11)$$

As can be found from the consideration the eq. (11), the maximum available gain is only achieved at the point where r_g (radius) equals zero , which is only obtained when the two-port is unconditionally stable. For the unconditionally stable case, the solution geometrically appears depending on the position of Z_{cgmax} which is the input impedance corresponding to the maximum available gain. It is obvious that if Z_{cgmax} is Region 1 the maximum achievable gain will have the value of a constant gain circle which is tangent to circle T_1. (Fig. 1) Similar considerations are made for the other solution regions and the corresponding terminations are also found by geometrical considerations in the Z_1-plane and the relation equations between Z_1 and Z_L, and the noise and Z_s.

In the conditionally stable case , the gain circles are constructed in the Z_i-plane in terms of the radius and center of the source plane stability circle , which equation is as follows: (Fig. 2)

$$G_T = \frac{r_{22}(1-|\rho_1|^2)}{|z_{12}|^2}\,(r_s^2-|Z_1+Z_{cs}|^2)\ /\ R_1 \qquad (12)$$

Similar considerations as in the case of unconditional stability are made to obtain the maximum gain and terminations.

5. DESIGN CONSIDERATIONS AND CONCLUSIONS:

A designer usually wants to follow the procedure given below for the design of low noise , low input VSWR transistor amplifiers: 1). Selection of transistor to meet requirements. 2). Modifying the requirements for a better and/or realisable performance. 3). Determination of corresponding terminations.
4). Realisation of matching blocks. The first two items are covered in Fig. 3, which are the maximum gain contours w. r. t noise and input VSWR. These curves not only show what is available, but let a designer trim the requirements at the expense or benefit of each other.

After performing the first two items of the design procedure given above, what maximum gain, noise and input VSWR will be obtained are known. From the geometrical considerations , one can calculate the input impedance and the corresponding load impedance. Then by drawing the particular required VSWR and noise circles in the source impedance plane , the wanted source impedance can easily be picked from the tangent or intersection points of both circles. It can also be found analytically.
The last item is a commonly known basic procedure and out of scope of this paper.

The conditional formulation of the maximum gain subject to noise and input VSWR may probably find substantial application in the preparation of data sheets of microwave transistors by manufacturers in the following years.

Because the maximum gain contours subject to noise and VSWR will not only be able to better characterize the performance of a low noise microwave transistor with the corresponding terminations, but will also help the user for a quick design. This formulation will . also let the designer be aware of some incompatible requirements of gain, noise and VSWR for narrow and medium band amplifier design.

Heretofore the common way followed in the design of a low noise amplifier is to get the noise figure as lower as possible while letting the input VSWR "go free" and use a circular or a balanced configuration to meet the input match requirement. If a circulator is used, its loss is added to the system noise figure. Besides , use of a circulator or a coupler with the MMIC's cancels the benefits of a small MMIC amplifier. This method is meant to be higher level tool in the design of MMIC amplifiers , with which we can overview the possible designs that we can supply to a network analysis program data for simple matching networks.

REFERENCES:

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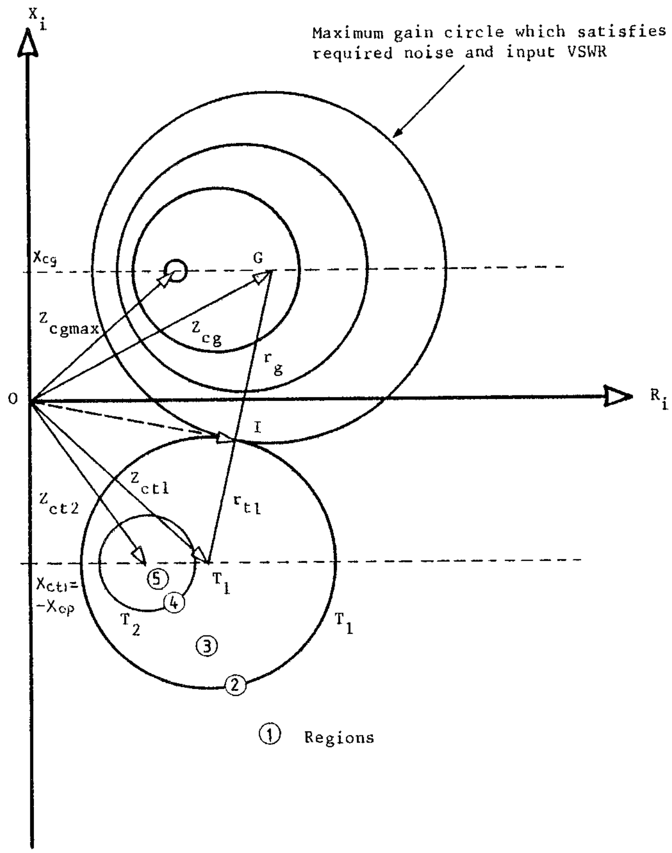


Fig.1 The solution of maximum gain and corresponding termination which satisfies the required noise and input VSWR

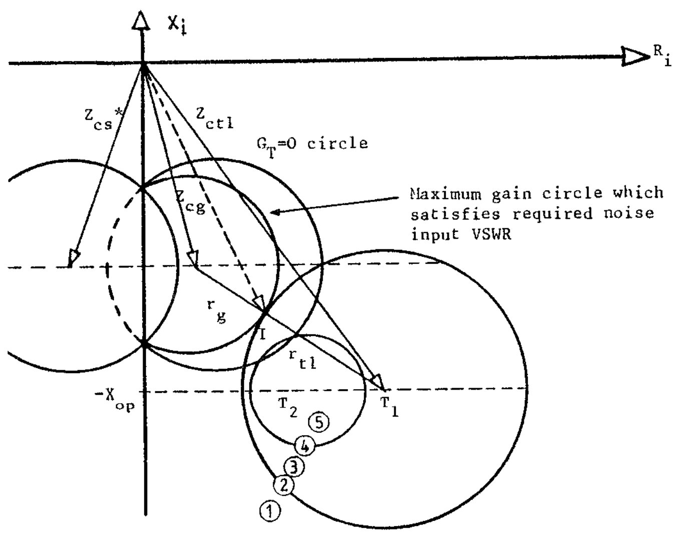
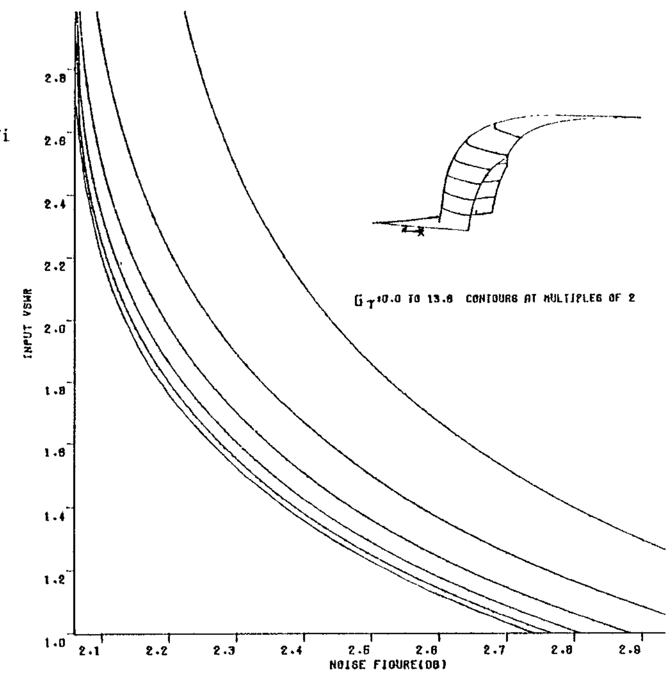


Fig. 2 Determination of maximum transducer power gain and corr. input impedance which satisfies required noise figure and input VSWR



G_T:10.0 TO 13.6 CONTOURS AT MULTIPLES OF 2

MAXIMUM TRANSDUCER POWER GAIN W.R.T. NOISE-FIGURE & INPUT VSWR AT 3.00 GHZ OF THE TRANSISTOR HXTR-6101

Fig.3